Exercise 79

What is the value of c such that the line $y = 2x + 3$ is tangent to the parabola $y = cx^2$?

Solution

We see that the slope of the tangent line is 2. Take the derivative of the equation for the parabola.

$$y' = \frac{d}{dx}(cx^2) = c\frac{d}{dx}(x^2) = c(2x) = 2cx$$

Set the derivative equal to 2 and solve for x.

$$y' = 2cx = 2 \quad \rightarrow \quad x = \frac{1}{c}$$

This is where the tangent line intersects the parabola; that is, the equations that represent them are equal when $x = 1/c$.

$$\text{At } x = \frac{1}{c}: \quad cx^2 = 2x + 3 \quad \rightarrow \quad c\left(\frac{1}{c}\right)^2 = 2\left(\frac{1}{c}\right) + 3$$

Solve this equation for c.

$$c = -\frac{1}{3}$$

Therefore, $y = 2x + 3$ is tangent to the curve $y = (-1/3)x^2$ when $x = -3$ as the following figure illustrates.